

Implications of factorization for the determination of hadronic form factors in $D_s^+ \rightarrow \phi$ transition.

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Abstract

Using factorization we determine the allowed domains of the ratios of form factors, $x = A_2(0)/A_1(0)$ and $y = V(0)/A_1(0)$, from the experimentally measured ratio $R_h \equiv \Gamma(D_s^+ \rightarrow \phi\rho^+)/\Gamma(D_s^+ \rightarrow \phi\pi^+)$ assuming three different scenarios for the q^2 -dependence of the form factors. We find that the allowed domains overlap with those obtained by using the experimentally measured ratio $R_{sl} = \Gamma(D_s^+ \rightarrow \phi\ell^+\nu_\ell)/\Gamma(D_s^+ \rightarrow \phi\pi^+)$ provided that the phenomenological parameter a_1 is 1.23. Such a comparison presents a genuine test of factorization. We calculate the longitudinal polarization fraction, $\Gamma_L/\Gamma \equiv \Gamma(D_s^+ \rightarrow \phi_L\rho_L^+)/\Gamma(D_s^+ \rightarrow \phi\rho^+)$, in the three scenarios for the q^2 -dependence of the form factors and emphasize the importance of measuring Γ_L/Γ . Finally we discuss the q^2 -distribution of the semileptonic decay and find that it is rather insensitive to the scenarios for the q^2 -dependence of the form factors, and unless very accurate data can be obtained it is unlikely to discriminate between the different scenarios. Useful information on the value of x might be obtained by the magnitude of the q^2 -distribution near $q^2 = 0$. However the most precise information on x and y would come from the knowledge of the longitudinal and left-right transverse polarizations of the final vector mesons in hadronic and/or semileptonic decays.

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Since the Cabibbo-Kobayashi-Maskawa (CKM)-favoured decays of the strange-charmed meson D_s^+ involve, at the quark level, $s\bar{s}$ pair, the vector meson ϕ emerges naturally as one of the decay products. Among its decay modes are the three most important ones, $D_s^+ \rightarrow \phi\pi^+$, $D_s^+ \rightarrow \phi\rho^+$ and $D_s^+ \rightarrow \phi\ell^+\nu_\ell$ ($\ell = e, \mu$) measured by several groups [1], and the topic of this paper.

The analysis of the experimental data for these modes involves three axial form factors, A_0 , A_1 and A_2 , and a vector form factor V , in the notation of Bauer, Stech and Wirbel (BSW) [2]. In spite of this fact which renders the analysis of experimental data involving the ϕ meson more complicated than the corresponding one with pseudoscalar mesons such as η or η' , it turns out that the polarization of the ϕ meson may provide crucial information that can be used to test the factorization assumption which is one of the basic ingredients in the computation of hadronic decay rates.

The fact that the π meson is light allows us to ignore the variation of the hadronic form factor $A_0^{D_s\phi}$ between $q^2 = m_\pi^2$, where it enters the description of $D_s^+ \rightarrow \phi\pi^+$, and $q^2 = 0$. This then allows us to eliminate $A_0^{D_s\phi}(0)$ in favour of $A_1^{D_s\phi}(0)$ and $A_2^{D_s\phi}(0)$ through [2],

$$2m_\phi A_0^{D_s\phi}(0) = (m_{D_s} + m_\phi)A_1^{D_s\phi}(0) - (m_{D_s} - m_\phi)A_2^{D_s\phi}(0) \quad (1)$$

Since the other decays, $D_s^+ \rightarrow \phi\rho^+$ and $D_s^+ \rightarrow \phi\ell^+\nu_\ell$, involve the form factors $A_1^{D_s\phi}$, $A_2^{D_s\phi}$ and $V^{D_s\phi}$, it proves convenient to factor out $A_1^{D_s\phi}(0)$ which then cancels in the ratio of the rates, and define the following ratios of the form factors at $q^2 = 0$,

$$x = \frac{A_2^{D_s\phi}(0)}{A_1^{D_s\phi}(0)} \quad y = \frac{V^{D_s\phi}(0)}{A_1^{D_s\phi}(0)} \quad (2)$$

In our analysis we do not adopt any of the several models of hadronic form factors for $D_s^+ \rightarrow \phi$ transition available in the literature to compute decay rates as such a procedure would not tell whether the factorization assumption or the model-form factors were being tested in the comparison between theory and experiment. Instead, we simply consider different scenarios for the q^2 -variation of the form factors (and do not make any assumptions about their absolute normalizations) and show that the measured ratio of the two hadronic rates,

$$R_h \equiv \Gamma(D_s^+ \rightarrow \phi\rho^+)/\Gamma(D_s^+ \rightarrow \phi\pi^+) \quad (3)$$

constrains the parameters x and y to an allowed domain of the x, y plane (limited by x and y non-negative as suggested by all phenomenological models). Such a domain can then be compared with the values of x and y already measured in the semileptonic modes [1, 3, 4]. We remind the reader that semileptonic data are analyzed with monopole q^2 -dependence for all form factors which is just one of the scenarios we discuss. A procedure such as ours that eschews reliance on a particular set of model-form factors produces not only a more bias-free test of factorization but also one that is independent of the BSW phenomenological parameter a_1 [2].

Further, since the ϕ meson is longitudinally polarized in $D_s^+ \rightarrow \phi\pi^+$, the ratio $\Gamma(D_s^+ \rightarrow \phi\pi^+)/\Gamma(D_s^+ \rightarrow \phi\rho^+)$ behaves physically like the longitudinal polarization fraction Γ_L/Γ in $D_s^+ \rightarrow \phi\rho^+$, where

$$\frac{\Gamma_L}{\Gamma} \equiv \frac{\Gamma(D_s^+ \rightarrow \phi_L\rho_L^+)}{\Gamma(D_s^+ \rightarrow \phi\rho^+)} \quad (4)$$

Therefore, there must be a link between these two quantities R_h and Γ_L/Γ which we explicitly demonstrate. Indeed, an upper and a lower limit for Γ_L/Γ can be calculated for the allowed x, y domain. A direct measurement of this quantity would provide a further test of factorization.

We now recall, for completeness, the various expressions for decay rates used in this paper though they can be found in published literature [5, 6, 7]. We start with the semileptonic decay $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$ in the zero lepton mass limit. Let us denote by q^2 the (effective mass)² of the $\ell \nu_\ell$ -system and introduce the following dimensionless quantities,

$$r \equiv \frac{m_\phi}{m_{D_s}} \quad t^2 \equiv \frac{q^2}{m_{D_s}^2} \quad (5)$$

with $0 \leq t^2 \leq (1-r)^2$.

In the D_s^+ rest system, the ϕ meson momentum $K(t^2)$ is given by,

$$K(t^2) = \frac{m_{D_s}}{2} k(t^2) \quad (6)$$

where $k(t^2) = [(1+r^2-t^2)^2 - 4r^2]^{1/2}$.

The t^2 -distribution for semileptonic decay is the sum of the contributions from the three polarization states,

$$\frac{d\Gamma}{dt^2} (D_s^+ \rightarrow \phi \ell^+ \nu_\ell) = \frac{G_F^2 m_{D_s}^5}{192\pi^3} |V_{cs}|^2 k(t^2) \{H_{LL}(t^2) + H_{++}(t^2) + H_{--}(t^2)\} \quad (7)$$

where

$$\begin{aligned} H_{LL}(t^2) &= \left[\frac{1+r}{2r}\right]^2 |(1-r^2-t^2)A_1^{D_s\phi}(t^2) - \frac{k^2(t^2)}{(1+r)^2} A_2^{D_s\phi}(t^2)|^2, \\ H_{\pm\pm}(t^2) &= (1+r)^2 t^2 |A_1^{D_s\phi}(t^2) \mp \frac{k(t^2)}{(1+r)^2} V^{D_s\phi}(t^2)|^2 \end{aligned} \quad (8)$$

For the hadronic modes where only the spectator diagram contributes in the factorization approximation, we obtain

$$\Gamma(D_s^+ \rightarrow \phi \pi^+) = \frac{G_F^2 m_{D_s}^5}{32\pi} |V_{cs}|^2 |V_{ud}|^2 a_1^2 \left(\frac{f_{\pi^+}}{m_{D_s}}\right)^2 k(0) H_{LL}(0) \quad (9)$$

where for $t = 0$,

$$k(0) H_{LL}(0) = (1-r^2)^3 \left[\frac{1+r}{2r}\right]^2 \left\{1 - \frac{1-r}{1+r} x\right\}^2 |A_1^{D_s\phi}(0)|^2 \quad (10)$$

For the $\phi \rho^+$ mode, first treating the ρ meson as a zero-width resonance, we get

$$\Gamma(D_s^+ \rightarrow \phi \rho^+) = \frac{G_F^2 m_{D_s}^5}{32\pi} |V_{cs}|^2 |V_{ud}|^2 a_1^2 \left(\frac{f_{\rho^+}}{m_{D_s}}\right)^2 k(t_\rho^2) \{H_{LL}(t_\rho^2) + H_{++}(t_\rho^2) + H_{--}(t_\rho^2)\} \quad (11)$$

where $t_\rho = m_\rho/m_{D_s}$. A comparison of (9) and (11) provides us with the precise connection between $\Gamma(D_s^+ \rightarrow \phi \pi^+)$ and $\Gamma(D_s^+ \rightarrow \phi_L \rho_L^+)$, the rate for the ρ meson in the longitudinally polarized state.

To take the finite ρ -width into account one has to smear the rate given in (11) over t^2 with a Breit-Wigner measure [7],

$$k(t_\rho^2) H_{\lambda\lambda}(t_\rho^2) \longrightarrow \frac{1}{\pi} \int_{\frac{4m_{D_s}^2}{m_{D_s}^2}}^{(1-r)^2} dt^2 \frac{\gamma_\rho t_\rho}{(t^2 - t_\rho^2)^2 + \gamma_\rho^2 t_\rho^2} k(t^2) H_{\lambda\lambda}(t^2) \quad (12)$$

where $\gamma_\rho = \Gamma_\rho/m_{D_s}$, Γ_ρ being the ρ -width.

The analysis of the experimental data requires certain assumptions for the q^2 -dependence of the hadronic form factors. In our analysis we assume the vector form factor, $V(q^2)$, to have a monopole q^2 -dependence with a pole mass $m_{1-} = 2.11 \text{ GeV}$ [2]. As for the axial form factors, $A_1(q^2)$ and $A_2(q^2)$, we investigate the following three scenarios :

Scenario I : Both $A_1(q^2)$ and $A_2(q^2)$ assumed to have monopole q^2 -dependence with pole mass $m_{1+} = 2.53 \text{ GeV}$ as in BSW [2].

Scenario II : Both $A_1(q^2)$ and $A_2(q^2)$ assumed to be independent of q^2 in the region of interest.

Scenario III : $A_1(q^2)$ assumed to be decreasing with q^2 ,

$$A_1^{D_s\phi}(q^2) = A_1^{D_s\phi}(0) \left[1 - \frac{q^2}{(3.5 \text{ GeV})^2} \right] \quad (13)$$

while $A_2(q^2)$ is assumed to have a monopole dependence with a pole mass of 3.5 GeV . This latter value was chosen just for illustration of a gently decreasing and increasing $A_1(q^2)$ and $A_2(q^2)$ respectively. Qualitatively, Scenarios II and III were inspired by the QCD sum rule calculations of Ref. [8] for $D \rightarrow K^*$ transition form factors.

Using the measured [9] hadronic ratio,

$$R_h = \frac{\Gamma(D_s^+ \rightarrow \phi\rho^+)}{\Gamma(D_s^+ \rightarrow \phi\pi^+)} = 1.86 \pm 0.26 \quad {}^{+0.29}_{-0.40} \quad (14)$$

we calculated the allowed domains in the x, y plane for each of the above scenarios. For $\Gamma(D_s^+ \rightarrow \phi\pi^+)$ we used expressions (9) and (10), and for $\Gamma(D_s^+ \rightarrow \phi\rho^+)$ the t^2 -smeared form of (11) in accordance with the prescription of Eq.(12).

In figs. 1, 2(a) and 2(b) we have plotted the boundary curves for scenarios I, II and III respectively. All positive values of x and y are allowed up to the boundary curve. Experiments [3, 4] on semileptonic decays, $D_s^+ \rightarrow \phi\ell^+\nu_\ell$, have determined x and y using the assumptions of scenario I. We have plotted their points in Fig. 1 too. We note that the point from E-653 [3] is excluded by the constraint set by R_h while the point from E-687 [4] has a small overlap with the domain determined from R_h . In our analysis the two errors in (14) were added in quadrature and we used the leptonic decay constants $f_{\pi^+} = 131.7 \text{ MeV}$ and $f_{\rho^+} = 212 \text{ MeV}$ in Eqs. (9) and (11).

Technically the curves for fixed R_h are hyperbolae in the x, y plane centered on the x axis whose principal axes are the x axis and an axis parallel to the y one. Of course only the part of the curve located on the left branch of the hyperbola has a physical meaning; the second branch requires too large a value of x , $x \geq 5.0$.

The general equation for these hyperbolae is,

$$\frac{(x - x_c)^2}{a^2} - \frac{y^2}{b^2} = 1. \quad (15)$$

We have listed the numerical values of x_c , a and b for the three models in Table 1 where we have also tabulated the upper limits of x and y , x_{max} for $y = 0$ and y_{max} for $x = 0$, and, of course, $x_{max} = x_c - a$.

SCENARIO	x_c	a	b	x_{max}	y_{max}
I	2.81	2.37	6.14	0.44	3.92
II	2.87	2.13	5.57	0.74	5.01
III	2.89	2.00	5.20	0.89	5.45

Table 1.

Parameters of the hyperbolae and x_{max} and y_{max} of the allowed x, y domain.

The full curves in Figs. 1, 2(a) and 2(b) use the smeared rate for $\Gamma(D_s^+ \rightarrow \phi \rho^+)$ in Eq.(12) while the dotted curves use the ρ^+ zero-width approximation in Eq.(11). We remark that for scenario I the allowed x, y domain for the ρ^+ zero-width approximation is empty. We emphasise that the corrections due to the non-zero ρ meson width play an important role in all scenarios.

Having established the range of x and y allowed by R_h , we can calculate using (11) and (12), the maximum and the minimum values of the longitudinal polarization fraction. We find the following limits;

$$\begin{aligned} \text{scenario I} & : & 0.43 \leq \frac{\Gamma_L}{\Gamma} & \leq 0.55 \\ \text{scenario II} & : & 0.36 \leq \frac{\Gamma_L}{\Gamma} & \leq 0.55 \\ \text{scenario III} & : & 0.33 \leq \frac{\Gamma_L}{\Gamma} & \leq 0.55 \end{aligned} \quad (16)$$

The upper limit occurs for $x = y = 0$ and is insensitive to the scenario adopted, while the lower limit depending on the maximum value of y , which occurs at $x = 0$, is rather scenario-dependent. In passing, we note that for a zero-width ρ -meson the maximum value of Γ_L/Γ is 0.51.

It is evident that an experimental measurement of the vector meson polarization in the decay $D_s^+ \rightarrow \phi \rho^+$ could provide important information on the parameters x and y . Further, a large experimental value of Γ_L/Γ exceeding unambiguously the upper bound given in (16) will be a clear indication of the failure of the factorization approximation. Equally, a measurement of Γ_L/Γ below 0.33 would rule out all the three scenarios assuming, of course, the validity of factorization approximation. In short, Γ_L/Γ could be used to discriminate between different scenarios.

In Figs. 1, 2(a) and 2(b) we have also plotted constant Γ_L/Γ curves (dashed-dotted) for three values, 0.3, 0.4 and 0.5 in order to exhibit the variation of Γ_L/Γ in the x, y plane. As previously, these curves are also hyperbolae of the type given by Eq.(15).

We now turn to a discussion of the semileptonic decays $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$. Experimentally, the following ratio has been measured [1, 4, 10, 11, 12]

$$R_{s\ell} \equiv \frac{\Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = 0.54 \pm 0.10 \quad (17)$$

Using (7) and (8) we can calculate $\Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell)$ for each of our three scenarios for the q^2 -dependence of the form factors. Then using (9) and (10) we can construct a theoretically a_1 -independent ratio,

$$a_1^2 \frac{\Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell)}{\Gamma(D_s^+ \rightarrow \phi \pi^+)} = a_1^2 R_{s\ell} \quad (18)$$

If a_1 were a well-determined parameter we could use (18) with $R_{s\ell}$ given by (17) to determine the allowed x, y domain from the ratio $R_{s\ell}$. It turns out that the limits of the allowed domains are rather sensitive to the value of a_1 used. If we treat a_1 as a free parameter we find that for $a_1 \approx 1.23$ the allowed domains obtained from R_h and $R_{s\ell}$ almost overlap. This is quite remarkable as the expected value of a_1 is 1.26 [13]. This we interpret as an evidence in favour of the factorization hypothesis in the hadronic decays.

We now address the question : What can we learn from the q^2 -distribution, $d\Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell)/dq^2$, of the semileptonic decays ?

For $0^- \rightarrow 0^- \ell \nu_\ell$ decay the answer is simple - we determine the q^2 -dependence of the form factor $F_1(q^2)$ as there is only one form factor involved. In the present case of $0^- \rightarrow 1^- \ell \nu$ decay the answer is not so clear as there are four terms in the q^2 -distribution associated with $A_1^2(q^2), A_1(q^2) \cdot A_2(q^2), A_2^2(q^2)$ and $V^2(q^2)$ and the only way to disentangle these four contributions, and then to measure the q^2 -dependence of the various form factors, is to determine simultaneously the longitudinal and left-right transverse polarizations of the final vector meson. We are far from achieving this in the case of $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$ decay in the near future.

Unfortunately, experimental measurement of the true q^2 -distribution of the semileptonic decays has not yet been possible because the momentum of the D_s^+ decaying in flight is not known [14] which, in turn, implies that the q^2 for an event is not accurately known. However, two experiments, E-653 [3] and E-687 [4] have determined the parameters x and y , assuming scenario I for the q^2 -dependence of the form factors, from the semileptonic decay $D_s^+ \rightarrow \phi \ell^+ \nu_\ell$. In the following we exploit this knowledge in absence of information on the q^2 -distribution $d\Gamma/dq^2$.

Let us introduce a unit normalized dimensionless quantity $X(t^2)$,

$$X(t^2) = \frac{1}{\Gamma} \frac{d}{dt^2} \Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell) \quad (19)$$

which depends on x and y , and on the type of q^2 -variation of the hadronic form factors. We first construct an "experimental" $X(t^2)$, denoted by $\overline{X}(t^2)$ in the following, using the experimental values of x and y obtained from a triple angular fit of the data [3, 4] in the scenario I. We then compute the same quantity, $X(t^2)$, for the values of x and y inside the allowed domains determined by us from R_h and shown in Figs. 1, 2(a) and 2(b). Finally we compare the $X(t^2)$ so constructed with $\overline{X}(t^2)$.

From Fig. 1 we note that the E-653 point lies well outside the allowed x, y domain while the E-687 point has a small overlap with this domain. Consequently we use only the latter data point in constructing $\overline{X}(t^2)$ and we adopt [4],

$$x = 1.1 \pm 0.81, \quad y = 1.8 \pm 0.92 \quad (20)$$

using scenario I, as was also the case in [4].

The resulting $\overline{X}(t^2)$ is shown in Fig. 3(a) in the form of nine curves corresponding to the central and one standard deviation extreme values of x and y . These nine curves break up naturally into three groups of three curves each, with each group being characterized by the value of x ; within each group the splitting due to different values of y is a much finer effect. Qualitatively at $t^2 = 0$, the group with the smallest x has the largest intercept with the vertical axis and within each group the curve with the smallest y has the largest intercept.

Our calculated $X(t^2)$ for scenarios I, II and III are shown in Figs. 3(b), 4(a) and 4(b) respectively in the form of three curves corresponding to the three extreme values of x and y for the allowed domains of Figs. 1, 2(a) and 2(b), $(x, y) = (0, 0), (0, y_{max}), (x_{max}, 0)$. All other choices of x and y in the allowed domain yield $X(t^2)$ which lies within the envelope of the three curves shown in Figs. 3(b), 4(a) and 4(b). The t^2 -distribution in these figures are then our prediction for $X(t^2)$ for x and y obtained from the hadronic ratio R_h . We observe that they depend strongly on x and very little on the q^2 scenarios. Therefore, we conclude that $X(t^2)$ cannot distinguish between different q^2 scenarios unless very precise measurements of the q^2 -distribution can be made which probably will not be the case in the near future. However, the determination of $X(t^2)$ near $t^2 = 0$ appears to be very useful for the determination of the parameter x .

We now compare our curves $X(t^2)$ with the "experimental" ones, $\overline{X}(t^2)$, of Fig. 3(a). For scenario I, we find some overlap between the curves of Fig. 3(a) and 3(b) provided that x is small. In particular the group of curves in Fig. 3(a) with $x = 0.29$ overlap well with the curve in Fig. 3(b) which uses $x = x_{max}$ and $y = 0$. The reason is obvious from Fig. 1 which shows that the points, $x = 0.29$ and y between 1 and 2 lie inside the allowed domain of x and y obtained from R_h . We remind the reader that $X(t^2)$ depends only weakly on y .

Such an overlap appears a little more problematic in scenarios II and III especially for large values of t^2 but cannot be excluded. For this reason we suggest to experimentalists to fit their triple angular distributions with different scenarios for the q^2 -dependence of the hadronic form factors for extraction of x and y . It would be interesting to see if the corresponding extracted values of x and y for scenarios II and III would be consistent with the allowed domains of x and y for these scenarios extracted from R_h and shown respectively in Figs. 2(a) and 2(b).

In conclusion, assuming factorization and using R_h (see(3) and (14) for definition), appropriately smeared over the ρ -width, we found the allowed domains in the x, y plane using three different scenarios for the q^2 -dependence of the form factors. These allowed domains plotted in Figs 1, 2(a) and 2(b) show sensitivity to the choice of the scenario. There is an overlap, though small, with the E-687 [4] values of x and y from semileptonic decay $\Gamma(D_s^+ \rightarrow \phi \ell^+ \nu_\ell)$ and the x, y allowed domain calculated by us from R_h in the relevant scenario - scenario I (See Fig. 1). This would suggest that the hadronic data

from [9] and the semileptonic data from [4] are consistent with the factorization hypothesis. E-653 [3] determination of x and y from semileptonic decay, on the other hand, does not support factorization assumption.

We have also determined the allowed domains of x and y from $R_{s\ell}$ (see (17) for definition) in the three scenarios. Such a determination is a_1 -dependent. We found that the two domains determined from R_h and $R_{s\ell}$ overlap almost completely for $a_1 \approx 1.23$ in remarkable agreement with the theoretically computed value of $a_1 = 1.26$ [13]. This fact supports the validity of the factorization assumption.

We have plotted the contours of constant Γ_L/Γ (see (4) for definition) in the x, y plane and calculated the maximum and the minimum values of Γ_L/Γ for the three scenarios of q^2 -dependence of the form factors considered by us. It is remarkable that the maximum value of $\Gamma_L/\Gamma = 0.55$ is independent of the scenario for the q^2 -dependence of the form factors, while the minimum value is scenario-dependent. Of course, a measurement of the longitudinal and left-right transverse polarizations will determine both x and y in a chosen scenario for the q^2 -dependence of the form factors.

A measurement of Γ_L/Γ could be important in testing factorization. For example, should the measured value of Γ_L/Γ exceed 0.55 unambiguously, it would signal a break down of the factorization hypothesis. A value below 0.33 would rule out all of our three scenarios for the q^2 -dependence of the form factors if factorization were to hold.

Finally, we suggest that experimentalists analyze their semileptonic data using for q^2 -dependence of the form factor forms other than monopoles to extract the value of x and y .

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Figure captions

1. **Fig. 1 :** The shaded region bounded by the solid line is the allowed domain of x , y from R_h within one standard deviation. The dashed-dot lines correspond to fixed values of Γ_L/Γ as indicated. All these curves are calculated with the scenario I for the q^2 dependence of form factors.
2. **Fig. 2(a) and 2(b) :** Same as Fig. 1, for scenarios II and III respectively. The dotted lines replace the solid lines in the zero width approximation of the ρ meson.
3. **Fig. 3 :**
 - (a) "Experimental" $\overline{X}(t^2)$ as explained in the text.
 - (b) Calculated $X(t^2)$ in the scenario I.
4. **Fig. 4 :**
 - (a) Calculated $X(t^2)$ in the scenario II.
 - (b) Calculated $X(t^2)$ in the scenario III.

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